# MATHEMATICS

**Paper 9709/01**
**Paper 1**

## General comments

Most candidates found the paper to be within their grasp. There were some excellent scripts, but at the same time, many from candidates who seemed unprepared for this level of work. There was little evidence that the candidates had had insufficient time to give of their best. It was very obvious that a large proportion of candidates were less than confident about three particular syllabus items: radian measure, unit vector and the notations f $'(x)$ and f $^{-1}(x)$.

## Comments on specific questions

### Question 1

This question presented many candidates with difficulty. Candidates were equally divided between using an algebraic or a calculus method. The algebraic method of eliminating $y$ (or $x$) from the equations and recognising that '$b^2 - 4ac$' = 0 for the resulting quadratic was the more successful, though errors in squaring '$2x + c$' were common. The calculus solution caused immediate problems over the taking of the root of '$4x$' and in the subsequent differentiation. Such errors as $y = 4x^{\frac{1}{2}}$ or $y = 2x^{-\frac{1}{2}}$ were common as was the answer $\frac{dy}{dx} = \frac{1}{2}(4x)^{-\frac{1}{2}}$. A minority of attempts obtained the answer $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$. Even when this was obtained, many candidates failed to realise the need to equate the gradient with 2, or were unable to solve for $x = \frac{1}{4}$, or failed to recognise the need to find $x$, $y$ and then $c$.

*Answer:* $\frac{1}{2}$.

### Question 2

This proved to be an easy question for most candidates, though a surprising number used the formula for the area under a curve, rather than the volume of rotation. Many candidates had problems in squaring $3x^{\frac{1}{4}}$, with $3x^{\frac{1}{2}}$ and $9x^{\frac{1}{16}}$ being common errors. The standard of integration and use of limits was very good.

*Answer:* $42\pi$.

### Question 3

This proved to be answered more successfully than similar questions in recent years and there were many excellent responses. The majority of attempts replaced $\tan^2 x$ by $\frac{\sin^2 x}{\cos^2 x}$ and then $\cos^2 x$ by $1 - \sin^2 x$, but there were many successful attempts which replaced $\tan^2 x$ by $\sec^2 x - 1$ and then $\sec^2 x$ by $\frac{1}{\cos^2 x}$. Most errors came in simplifying the left hand side to $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$, though once this was achieved, candidates usually arrived at the correct answer.

**Question 4**

Answers varied considerably, with many candidates producing excellent solutions, and others failing to get started. Most candidates recognised that the equation was a quadratic in either $x^2$ or $x^{-2}$, and introduced another variable. Some replaced $x^2$ by $x$ and failed to adjust at the end. Replacing $y$ by $x^{-2}$, and then taking $y$ to be $x^2$ at the end was also a common error. At least half of all attempts failed to realise that there are two solutions to the equation $x^2 = k$. The most depressing error came from the many candidates who stated that '$x^2(4x^2 - 1) = 18$' implied that '$x^2 = 18$ or $4x^2 - 1 = 18$'.

*Answer:* $x = \pm 1.5$.

**Question 5**

This question was generally well answered with the majority of solutions recognising the need to use trigonometry in triangle *OAX*. The majority used 'tangent' correctly but a significant proportion still failed to understand the meaning of the word 'exact'. Using a decimal value from a calculator and attempting to express as a multiple of √3 cannot gain full marks. In part **(ii)**, most candidates realised the need to subtract the area of a sector from the sum of the areas of triangles *OAX* and *OXB.* Again, however, a few ignored the request to express the answer in terms of √3 and $\pi$.

*Answers*: **(i)** $4\sqrt{3}$ ; **(ii)** $48\sqrt{3} - 24\pi$.

**Question 6**

This proved to be a good source of marks for most candidates. In part **(i)**, the majority of candidates recognised that the product of the gradients of *AB* and *BC* was equal to −1. Errors in calculating the gradient of *AB* were rare and most candidates obtained a correct equation for the line *BC*. Although a small minority set $x = 0$ instead of $y = 0$, the majority obtained the coordinates of *C* correctly. A minority of candidates however realised that the coordinates of *D* could be written down by using the fact that $\overrightarrow{BA}$ was equal to $\overrightarrow{CD}$. Surprisingly, even after the lengthy calculations of the equations of *AD* and *CD*, most candidates obtained correct values for the coordinates of *D*.

*Answers*: **(i)** $3y + 2x = 20$ ; **(ii)** $C(10, 0)$, $D(14, 6)$.

**Question 7**

Most candidates were able to write down two correct equations for $a$ and $r$, though in many cases $a = 3$ was used along with the equation for the sum to infinity. The solution of the resulting quadratic equation was generally accurate and many candidates obtained full marks for part **(i)**. In part **(ii)**, a few candidates found the sum of a geometric, instead of an arithmetic progression but the common error was the failure to realise that the common difference equalled '$3 - a$'.

*Answers*: **(i)** 6 ; **(ii)** −450.

**Question 8**

This was poorly answered showing a poor understanding of radian measure. Less than a half of all candidates were able to evaluate $a$ and $b$, mainly through failure to recognise that $\cos(\pi) = -1$. In part **(ii)**, the majority of candidates realised the need to find $2x$ first, but even when $a$ and $b$ had been correctly evaluated, answers to this part were almost always given in degrees. A small minority of solutions were correct. The graphs in part **(iii)** were also poorly drawn, with a large proportion showing curves that failed to flatten out at $x = 0$ and at $x = \pi$, whilst others were triangular in shape.

*Answers*: **(i)** $a = 3$, $b = -4$ ; **(ii)** $x = 0.361$, $2.78$ .

**Question 9**

The question was reasonably answered with almost all candidates able to find $\overrightarrow{AB}$ and most calculating $\overrightarrow{OC}$ correctly. There were, however, many errors in sign caused by the inability to evaluate either (−4) − (−2) or (−4) + (−2) correctly. Again the majority of candidates failed to understand the meaning of 'unit vector', with the position vector $\overrightarrow{OC}$ being taken as the unit vector in most cases. Part **(ii)** was well answered, with the majority of candidates realising that equating coefficients in the **i** and **j** directions led to two simultaneous equations for *m* and for *n*, and equating coefficients in the **k** direction led to the value of *k*.

*Answers*: **(i)** $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$; **(ii)** $m = -2$, $n = 3$, $k = -8$.

**Question 10**

This question was very well answered, showing that most candidates had a good understanding of the basic techniques of calculus. Part **(i)** was nearly always correctly answered. In part **(ii)**, most candidates used and solved $\frac{dy}{dx} = 0$, though there were several scripts in which the second differential was set to zero. Surprisingly in part **(ii)**, a lot of candidates misread 'coordinates' for 'coordinate' and failed to calculate the value of *y* at the stationary point. Most candidates realised that the stationary point was a minimum point, usually by consideration of the sign of the second differential. Part **(iii)** was also well done, though numerical errors were made when $x = -2$ was substituted into $\frac{-1}{\frac{dy}{dx}}$ or into the equation of a line. Part **(iv)** was reasonably done, though the integration of $\frac{8}{x^2}$ presented many weaker candidates with problems.

*Answers*: **(i)** $2 - \frac{16}{x^3}$, $\frac{48}{x^4}$; **(ii)** (2, 6), Minimum; **(iv)** 7.

**Question 11**

Many candidates confused parts **(i)** and **(ii)** through failure to understand the difference between $f'(x)$ and $f^{-1}(x)$. The differentiation in part **(i)** was generally accurate, though a significant number of candidates failed to recognise that the function was composite and omitted the '×2'. The proof that the function was decreasing was poorly done, with most candidates believing it sufficient to show that $f'(x)$ was negative at one rather than *all* values of *x*. The majority of candidates made a pleasing attempt at forming both $f^{-1}(x)$ in part **(ii)** and fg(*x*) in part **(iv)**. Whilst part **(iv)** was nearly always correctly answered, it was very rare to see a correct domain for $f^{-1}(x)$ in part **(ii)**. The sketch graphs in part **(iii)** were poorly drawn with many attempts at $y = f^{-1}(x)$ failing to stop on the *x*-axis and many others not being a decreasing function. Most candidates did, however, realise that the graph of $y = f^{-1}(x)$ was a reflection of $y = f(x)$ in the line $y = x$.

*Answers*: **(i)** $\frac{-12}{(2x+3)^2}$; **(ii)** $\frac{3}{x} - \frac{3}{2}$, $0 < x \leqslant 2$; **(iv)** $x = 1$.

# MATHEMATICS

**Paper 9709/02**

**Paper 2**

## General comments

Candidates' performance varied considerably. Many had been well prepared and showed considerable confidence in their responses. However, a large minority failed to score in double figures and often these candidates failed to use correctly the laws and results of the calculus. Many failed, for example, to appreciate that the derivative of a product of two functions of $x$ consists of two, not one, terms. In attempting to integrate candidates often used $\int [f(x)]^n \mathrm{d}x = \frac{1}{n+1}[f(x)]^{n+1}$, for example writing $\int \cos^2 x \, \mathrm{d}x = \frac{1}{3}\cos^3 x$.

In general, the standard of presentation was often poor and there was much work that was scrappy, showing little attempt at attractive presentation. The Examiners were concerned by the large number of candidates dividing each page into two columns (and leaving the reverse side of the paper bank); work was crammed into a jumbled page with solutions to two or three questions often overlapping. Centres are encouraged to ensure that their candidates do not adopt this practice which makes marking very difficult.

In many cases, there was little evidence that past papers have been worked through by candidates, or that the comments in previous Reports on the Examination had been heeded thereby reducing the occurrence of common false methods and techniques among those being prepared for the examination.

## Comments on specific questions

### Question 1

This question was well attempted for the most part, with many candidates scoring the first three or all four marks. Very weak candidates simply set $3 - x = x + 2$, and hence $x = \frac{1}{2}$ was obtained. Many even set $x - 3 = x + 2$ and then struggled to find a corresponding value of $x$. Few used graphical techniques, and these were invariably excellent solutions. Most candidates opted to square each side and compared $(x^2 - 6x + 9)$ with $(x^2 + 4x + 4)$, yielding a linear equation (or inequality) in $x$, though may failed to tidy up correctly terms in $x$ and/or the constants. Those using inequalities were surprisingly good at handling $-10x > -5$; few candidates deduced that $x > \frac{1}{2}$, though a few thought that $x < 2$.

*Answer*: $x < \frac{1}{2}$.

### Question 2

**(i)** Many candidates scored poorly as they never stated or implied that $y\ln 3 = (x + 2)\ln 4$ was a straight line of the form $ay = bx + c$ and never stated the gradient, or gave it an approximate value rather than the exact value requested.

**(ii)** Poor arithmetic was evident in many solutions, and the use of an approximate gradient from part **(i)**, usually 2.62, affected the 2nd decimal place in the $x$-coordinate of intersection. Many solutions were ruined by the initial supposition that $y\ln 3 = x + 2\ln 4$, rather than $(x + 2)\ln 4$.

*Answers*: **(i)** $\frac{\ln 4}{\ln 3}$; **(ii)** 3.42 .

**Question 3**

**(i)** Nearly all candidates obtained the value of $\frac{dy}{dt}$. Better candidates used $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$, but a majority tried to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dx}{dt}$ and many were unable to obtain $\frac{dt}{dx}$ from $\frac{dx}{dt}$, e.g. after writing $\frac{dx}{dt} = 3 + \frac{1}{t-1}$ gave $\frac{dt}{dx} = \frac{1}{3} + (t-1)$.

**(ii)** Some candidates obtained the correct quadratic and eventually scored full marks for this part. However, those who obtained an erroneous quadratic (usually after an error in part **(i)**) failed to make further progress.

*Answers*: **(i)** $\frac{2t(t-1)}{3t-2}$; **(ii)** (6, 5).

**Question 4**

**(i)** The –20 was often missed when finding f(–2), but this was generally a well-attempted part.

**(ii)** A large number of candidates started to divide by $(x^2 - 4)$, then crossed out their attempt only to do something else, e.g. divide by $(x - 2)$ or $(x + 2)$, or both of these in succession. Given that most candidates may be assumed to be competent in numerical long division, the Examiners were surprised at how few correct divisions of candidates' p($x$) by $(x^2 - 4)$ were seen.

*Answers*: **(i)** –3, 2; **(ii)** $5x - 10$.

**Question 5**

**(i)** A disappointingly high number of candidates could not sketch y = 3 – $x$ correctly, and very few could make a reasonable attempt at sketching $y = \sec x$.

**(ii)** Many candidates still do not seem to understand what is required for this sort of question, namely, if f($x$) is defined by $f(x) = \sec x + x - 3$, to calculate f(1.0) and f(1.2) and show that they are of different sign. The change of sign then indicates that $y = \sec x + x - 3$ crosses the $x$-axis between $x = 1.0$ and x = 1.2 and hence there is a root of $\sec x = 3 - x$ between those two values of $x$.

**(iii)** Most candidates made a good attempt at this, sometimes proving the result in reverse, showing that $x = \cos^{-1}\left(\frac{1}{3-x}\right)$ reduces to $\sec x = 3 - x$.

**(iv)** Many candidates did not work to 4 decimal places in their iterations as requested. A substantial minority thought that $x$ was measured in degrees, and obtained 90.65°.

*Answer*: **(iv)** 1.04 .

**Question 6**

**(i)** Although many candidates knew that one form for $\cos 2x$ is $2\cos^2 x - 1$, few could convert this to $\cos^2 x = \frac{1+\cos 2x}{2}$ and invariably made little progress in this part.

**(ii)** Where the form was correct in part **(i)**, most obtained full marks here. Others simply could not integrate, e.g. '$\int \cos^2 x \, dx = \frac{1}{3}\cos^3 x$' or '$\frac{\cos^3 x}{3\sin x}$' was in evidence among many other variations.

**(iii)** Many candidates correctly stated that $\sin^2 x = \frac{1-\cos 2x}{2}$ and proceeded to score at least 2 marks. Others used $\int \sin^2 x\,dx = \int 1\,dx - \int \cos^2 x\,dx$ and the result from part **(ii)**.

*Answers*: **(i)** $\frac{1}{2}+\frac{1}{2}\cos 2x$ ; **(iii)** $\frac{1}{6}\pi - \frac{1}{8}\sqrt{3}$ .

**Question 7**

**(i)** This part managed to confuse a sizeable minority.

**(ii)** Many candidates obtained only one term for $\frac{dy}{dx}$, usually $-e^x \sin x$.

**(iii)** Many candidates worked to insufficient accuracy. Others used $x$-values as the $y$–ordinates and a large number had 4, or even 2, strips.

**(iv)** The reasoning used was generally poor, and few candidates pointed out that small areas occurred between the tops of the trapezia and the curve. Many gave their explanation in terms of one trapezium.

*Answers*: **(i)** (0, 1); **(ii)** $\frac{1}{4}\pi$; **(iii)** 1.77; **(iv)** Underestimate.

# MATHEMATICS

**Paper 9709/03**

**Paper 3**

## General comments

The standard of work varied widely. No question appeared to be of unreasonable difficulty and candidates seemed to have sufficient time. The questions that were done particularly well were **Question 2** (algebra) and **Question 9** (vector geometry). Those that were done least well were **Question 5** (trigonometry), **Question 7** (integration) and **Question 8** (complex numbers).

In general the presentation of work is good but there remain two respects in which it is sometimes unsatisfactory. Firstly there are still a few candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs most frequently when they are working towards answers given in the question paper, for example as in **Question 7**. Examiners penalize the omission of essential working in such questions.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

## Comments on specific questions

### Question 1

Though a few attempted to expand directly, most candidates took out a factor of $\frac{1}{4}$ and expanded $\left(1+\frac{3}{2}x\right)^{-2}$. Apart from slips in simplifying the coefficients, the main mistakes were the use of incorrect numerical factors, typically 2 and $\frac{1}{2}$.

*Answer*: $\frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2$.

### Question 2

This was very well answered by a variety of methods. In part **(ii)** it was quite common for the correct quadratic factor to appear in the working, for example as part of a factorisation of p($x$) or solution of p($x$) = 0, or as the quotient in a division, yet the quadratic factor was never explicitly identified as such. This suggests that the term 'factor' is not fully understood by some candidates.

*Answers*: **(i)** 4; **(ii)** $x^2-2x+2$.

**Question 3**

This question differentiated well. Nearly all candidates knew and used the product rule, but the main sources of error were (*a*) incorrect differentiation of sin 2*x*, (*b*) the use of degrees instead of radians, (*c*) taking the gradient of the tangent to be the negative reciprocal of the gradient of the curve, and (*d*) using the general gradient instead of the gradient at the point when forming the equation of the tangent.

*Answer*: $y = x$.

**Question 4**

Most candidates were able to reach and solve the equation $u^2 - 2u - 1 = 0$. Some stopped at this point, but the majority knew how to calculate *x* from a positive value of *u* and rejected negative values of *u*. The error of working with the prematurely rounded value $u = 2.41$ caused a significant number to lose the final mark. Routine checking of one's work would have benefited those candidates who made a sign error and reached $u^2 - 2u + 1 = 0$, for the solution $u = 1$ leads to $x = 0$ and substitution in the original equation gives $1 = 2 + 1$. The error of thinking $\ln(a + b)$ equalled $\ln a + \ln b$ was seen when candidates were calculating the logarithm of $1+\sqrt{2}$. This same misconception was also evident when candidates took logarithms of both sides of the original equation and reached an erroneous linear equation in *x*.

*Answer*: 0.802.

**Question 5**

Part **(i)** was generally well answered, though some candidates failed to give the exact value of $\alpha$ and the trigonometric work was not always sound. Part **(ii)** was rarely answered correctly. The simple step of replacing the reciprocal of the squared cosine by the squared secant converts the integrand into a recognizable standard form, but not many candidates realised this. Those that did usually went on to score full marks, though occasionally some lost the final mark because they failed to give sufficient working to justify the given answer.

*Answer*: **(i)** $2\cos\left(\theta - \frac{1}{3}\pi\right)$.

**Question 6**

**(i)** Candidates who took the area of triangle *AOB* to be $\frac{1}{2}r^2\sin\alpha$ usually made short work of this problem. The remainder either omitted this part altogether or struggled to set up a correct equation and reduce it to the given form.

**(ii)** Some candidates seemed to believe that a statement involving 'positive' and 'negative' was sufficient, without any reference to there being a change of sign, or even the function under consideration. However others did make clear the function they were considering and evaluated numerical values as required, before stating what the change of sign meant.

**(iii)** This part was generally well answered.

**(iv)** Most candidates gave the result of each iteration to 4 decimal places as required, though some failed to give the final answer to 2 decimal places. Those who calculated in degree mode obtained 0.64188… as first iterate. The fact that this differs considerably from the initial value of 1.8 and also lies outside the interval stated in part **(ii)** should have been a warning that something was wrong. However such candidates invariably went on iterating and wasted valuable time on fruitless work.

*Answer*: **(iv)** 1.90.

**Question 7**

Though there were many completely sound answers to part **(i)** there were also many attempts which omitted key steps or were simply incorrect. The main sources of error were (*a*) an incorrect relation between d*u* and d*x*, and (*b*) failure to replace both *x* and d*x* throughout the integral. Part **(ii)** was poorly answered. Many candidates failed to realise the need for partial fractions. Those who did usually integrated successfully but did not always give sufficient working to justify the given answer.

**Question 8**

This question was not answered well. Elementary slips marred many attempts at expressing *u* in the form $x + iy$. Careful checking of working would have saved loss of marks later. Those who obtained $-1 - i$ usually found the correct value of mod *u* but tended to give arg(*u*) the incorrect value $\frac{1}{4}\pi$. There was a good understanding of the method for finding the modulus and argument of $u^2$ from those found for *u*. However candidates who chose to first obtain $u^2$ in Cartesian form were again prone to make errors in either deriving the form or finding the argument of the result. Few completely correct sketches were seen in part **(ii)**, partly because *u* and/or $u^2$ were wrong, or because the candidate could not see the need for a circle of radius 2 and centre at the origin as well as the perpendicular bisector of the line joining the points representing the two plotted points. Sometimes an otherwise correct sketch lost the final mark because the candidate shaded the unwanted segment of the circle.

*Answers*: **(i)** $\sqrt{2}$ and $-\frac{3}{4}\pi$, 2 and $\frac{1}{2}\pi$.

**Question 9**

Examiners reported that part **(i)** was answered confidently and well by a variety of methods. The use of a vector product was popular. Part **(ii)** was answered less well. Some candidates were unable to find a relevant pair of vectors from which to calculate the required angle. Those who correctly decided to work with vectors normal to the planes seemed to feel the need to calculate a vector normal to the *x*-*y* plane *OAB* instead of simply taking it to be the unit vector **k**. This calculation was sometimes incorrect and accuracy marks were needlessly lost.

*Answers*: **(i)** $4x + 2y + z = 8$; **(ii)** 77.4°.

**Question 10**

This question was quite well answered. There were many correct solutions to part **(i)**. Candidates who merely verified that the boundary conditions satisfied the given answer scored zero. Most candidates separated variables correctly in part **(ii)** but a sign error when integrating $(9-h)^{-\frac{1}{3}}$ was quite common. This error might have been corrected if the derivative of $\frac{3}{2}(9-h)^{\frac{2}{3}}$ had been examined. The calculation of a constant of integration was usually done well. The subsequent rearrangement of the particular solution to make *h* the subject proved testing for some candidates. They found it difficult to manipulate an expression involving a fractional index correctly. Parts **(iii)** and **(iv)** were done well by the strongest candidates, though in part **(iv)** some used an inappropriate proportional argument leading to $t = 30$, instead of substituting $h = 4.5$ in one of the forms of their particular integral.

*Answers*: **(ii)** $h = 9 - \left(4 - \frac{1}{15}t\right)^{\frac{3}{2}}$; **(iii)** 9 m, 60 years; **(iv)** 19.1 s.

# MATHEMATICS

**Paper 9709/04**

**Paper 4**

## General comments

The paper was generally well attempted with a significant number of candidates scoring high marks. However it is disappointing to report that some candidates scored very low marks, and were clearly not ready for examination at this level.

## Comments on specific questions

### Question 1

**(i)** This part is a routine exercise on the use of $v^2 = u^2 + 2as$ (or $\frac{s}{t} = \frac{u+v}{2}$ and $v = u + at$). It was answered correctly by almost every candidate.

**(ii)** Although this was intended as a routine application of $a = g \sin|\alpha|$ the formula was almost entirely unused. However it is pleasing to note that some candidates used Newton's second law, or more commonly the principle of conservation of energy, in an appropriate way. Generally, however, this part of the question was omitted or very poorly attempted.

*Answers*: **(i)** 0.5 $\text{ms}^{-2}$; **(ii)** 2.9.

### Question 2

This question was intended to test the understanding of the concepts of component and resultant. It was poorly attempted, demonstrating a widespread lack of the required understanding.

**(i)** Although a significant number of candidates thought that $\theta$ is 45, most thought that $\theta°$ is the angle opposite the side of length 9 in a triangle of sides 8, 8 and 9. There were relatively few correct answers.

**(ii)** This part was better attempted than part **(i)**, many candidates using $R^2 = (8 + 8\cos\theta)^2 + (8\sin\theta)^2$, albeit with an incorrect value of $\theta$ in most cases. A significant proportion of candidates effectively assumed that $\theta$ is 90, and used $R^2 = 8^2 + 8^2$.

*Answers*: **(i)** 82.8; **(ii)** 12 N.

### Question 3

**(i)** This part was well attempted and most candidates scored all 3 marks. However some candidates who correctly found the driving force to be equal to 600 N did not proceed to equate this quantity with the resistance to motion and hence state that $R = 600$.

**(ii)** This part was also well attempted, although very many candidates obtained the answer 0.75 $\text{ms}^{-2}$, taking no account of the resistance to motion.

*Answers*: **(i)** 600; **(ii)** 0.25 $\text{ms}^{-2}$.

**Question 4**

**(i)** This part was well attempted and many candidates scored full marks. However some candidates did not find the tension as required, in an otherwise complete and correct solution. Candidates who tried to write down an equation in $a$ only usually obtained $0.6g - 0.2g = 0.6a$ leading to the incorrect $a = 3.33$. Candidates who used energy considered only $P$, rather than both $P$ and $Q$, and did not realise that they would therefore need to take account of the work done on $P$ by the tension.

**(ii)** This part was well attempted, although a significant minority of candidates used $s = \frac{1}{2}gt^2$ instead of $s = \frac{1}{2}at^2$.

*Answers*: **(i)** 5 $\text{ms}^{-2}$; **(ii)** 0.6 s.

**Question 5**

**(i)** Most candidates found the increase in kinetic energy correctly, although a few used $\frac{1}{2}12\,500(25 - 17)^2$. Among those candidates who obtained 2 100 000 J some proceeded to the correct answer 7 100 000 J (or 7100 kJ), some proceeded to 2 105 000 J, mixing the units of the kinetic energy (KE) found and the given 5000 kJ, some proceeded to 2 900 000 J (or 2900 kJ), subtracting the KE instead of adding, and some did not proceed beyond the calculation of the KE.

**(ii)** Almost all candidates considered the work-energy for the motion between $B$ and $C$ (rather than between $A$ and $C$), requiring the change in kinetic energy to be represented in the resulting equation.

It was unusual to see the work-energy equation represented by all four of its components. Although the kinetic energy and potential energy were almost always represented, one or both of the work done by the driving force and the work done by the resistance were often omitted. When the work done by the resistance was represented, this was often by just 4800 instead of $4800 \times 500$.

*Answers*: **(i)** 7100 kJ; **(ii)** 24 m.

**Question 6**

This question was well attempted by those candidates who realised the need to use the calculus. However very many candidates omitted the question or scored no marks. Irrelevant use of equations relating to constant acceleration was common among candidates in the latter category.

**(i)** Many candidates integrated $v(t)$ and applied limits correctly to obtain $s(10) = 50$. Some candidates recognised that the quadratic function $v(t)$ is symmetric about $t = 5$ and found $s(t) = 25$ instead. Candidates who approached the next stage by using the area property for the $t$-$v$ graph obtained $v_{max}$ correctly, from either $\frac{1}{2}10v_{max} = 50$ or $\frac{1}{2}5v_{max} = 25$. Candidates who used $\frac{s}{t} = \frac{u+v}{2}$ also obtained $v_{max}$ correctly from $\frac{25}{5} = \frac{0 + v_{max}}{2}$. However many candidates obtained the answer 10 fortuitously using this formula, from $\frac{50}{10} = \frac{0 + v_{max}}{2}$, which is clearly an inappropriate use because $P$'s acceleration is not the same throughout the whole 10 seconds of its motion. Another common method of obtaining the given answer incorrectly was to use the formula $v = \frac{s}{t}\left(= \frac{50}{5}\right)$.

**(ii)** Most candidates who attempted this part of the question found $a(t)$ correctly for $Q$. Thereafter very many equated this to 2 and proceeded to find the correct answer. However many others equated $a(t)$ to zero, or to some other number or to a function of $t$, making no progress.

*Answer*: **(ii)** $1\frac{2}{3}$.

**Question 7**

**(i)** Many candidates were successful in obtaining the correct answer for $T$. However some of these candidates did not go beyond this stage and many who did had a term missing when resolving forces vertically. Those who omitted the weight of the block often obtained a resultant force of magnitude 150 N which was sometimes given as the answer for the magnitude of the contact force. More often it was given as $Y$, the vertical component of the contact force, there being an expectation that there is also a (non-zero) horizontal component, despite the fact that the surface $AB$ is smooth. Those who omitted the contact force usually obtained a second value for $T$, having already found the correct value.

**(ii)** Most candidates did not resolve forces horizontally, but carried forward their value of $T$ from part **(i)**. Those candidates who did resolve forces horizontally made errors. These errors included using the value of $T$ from part **(i)**, representing the frictional force twice, once as 25 and once as $\mu R$, or writing the frictional force as $25\mu$. Those who did not resolve forces horizontally usually did so vertically, using the value of $T$ from part **(i)**. As in part **(i)** many candidates had a term missing. Almost all candidates made some attempt to obtain $\mu$ from $F = \mu R$, but a very large proportion of candidates used a value of $F$ different from the given 25. The majority of candidates used a value of $R$ obtained in part **(i)**.

*Answers*: **(i)** 130, 50 N; **(ii)** 0.268.

# MATHEMATICS

**Paper 9709/05**
**Paper 5**

## General comments

This paper proved to be a fair test for any candidate with a clear understanding of basic mechanical ideas. The majority of candidates had sufficient time to attempt all the questions on the paper.

Most candidates worked to the required accuracy and very few examples of premature approximation were seen. Nearly all candidates used the specified value of $g$.

Again it is necessary to stress the need for candidates to use good, clear diagrams on their answer sheets in order to aid their solutions. It was pleasing to see more candidates using diagrams this year.

**Questions 1**, **4(i)**, **6(iii)** and **7(iii)** were generally found to be the more difficult questions on the paper.

## Comments on specific questions

### Question 1

This was intended to be a straightforward start to the paper but there were many poor attempts.

**(i)** Although the formula for the centre of mass of a sector lamina is quoted on the formula sheet, not many candidates used the correct one. $\frac{3r}{8}$ and $\frac{r\sin\alpha}{\alpha}$ were often seen.

**(ii)** Most candidates used $v = r\omega$ but simply put $r = 5$ instead of the value found in part **(i)**.

*Answers*: **(i)** 2.12 m; **(ii)** 8.49 $\text{ms}^{-1}$.

### Question 2

**(i)** $x = \frac{3}{2}$ was usually seen. Occasionally $x = \frac{2}{3}$ was calculated.

**(ii)** This question was generally well attempted, but quite a number of candidates simply stated $v = \int(3-2x)\,dx$ instead of using $a = v\frac{dv}{dx} = 3 - 2x$ and then separating the variables and integrating.

*Answers*: **(i)** 1.5; **(ii)** 2.12 $\text{ms}^{-1}$.

### Question 3

This question was generally well done, but many candidates mixed up the sines and cosines. Some candidates even introduced an angle of 45°. The idea of resolving horizontally and vertically was often recognised and attempted. Occasionally $R = mg\cos\theta$ was used.

*Answers*: 1.10, 0.784.

**Question 4**

**(i)** Some common errors were (*a*) to use the distance of the centre of mass from *A* to be 0.2 or 0.3, (*b*) to consider the two components of *T* but to only use one of them when taking moments and (*c*) to use angle *A* as 45°. Not too many candidates realised that the moment of *T* about *A* was simply *T* x 1. Some candidates simply tried to resolve instead of taking moments.

**(ii)** Many candidates used a correct method to answer this part.

*Answers*: **(i)** 16N; **(ii)** 12.8N, 30.4N.

**Question 5**

**(i)** A common error here was to consider the loss in gravitational potential energy to be 0.8 x *g* x 0.1 instead of 0.8 x *g* x (0.5 + 0.1).

**(ii)** Some candidates stated $140x^2 = 4.8$ (or some other numerical value) instead of $140x^2 = 0.8g(0.5 + x)$.

*Answers*: **(i)** 2.92 $ms^{-1}$; **(ii)** 0.2 m.

**Question 6**

**(i)** Many candidates scored full marks on this part of the question.

**(ii)** $a = 20\ ms^{-2}$ was often seen. Candidates getting this part wrong often used $T_1 + T_2 = ma$ or they considered the strings to be in a vertical plane when the question clearly states that they are on a smooth table.

**(iii)** Not many candidates arrived at the correct answer. Some candidates tried to use an energy equation instead of simply putting the tensions equal. Of those who knew to equate tensions many could not work out the correct extensions in the strings and so had the wrong equation. Again some candidates considered the strings to be in a vertical plane. This part of the question proved to be demanding for many of the candidates.

*Answers*: **(i)** 26N, 7N; **(ii)** 20 $ms^{-2}$; **(iii)** 0.933 m.

**Question 7**

**(i)** This part was often well done.

**(ii)** Candidates often scored well on this part.

**(iii)** Very few candidates managed to solve this part of the question. Often the horizontal and vertical components of the velocity at the point where the particle made an angle of 20° were seen as 65cos20° and 65sin20°. Other candidates tried to use $\tan 20° = \frac{y}{x}$ instead of $\tan 20° = \frac{v_y}{v_x}$ while some tried to use the path of the trajectory of the particle. Some candidates were able to work out $t = 5.09$ but thought that this was the answer – not realising that it was the time to reach the first point when the particle was at 20° to the horizontal. Unfortunately many candidates did not use a correct complete method to answer this part of the question.

**(iv)** Candidates often scored the 1 mark for using 65sin67.4° x their time from part **(iii)**.

*Answers*: **(i)** 67.4°; **(ii)** 180 m; **(iii)** 1.82 s; **(iv)** 45.5 m.

# MATHEMATICS

**Paper 9709/06**

**Paper 6**

## General comments

This paper again produced a wide range of marks. Some candidates omitted to put any units in the stem-and-leaf key in **Question 4**. Whilst not usually penalising omission of units, it was considered that in a data representation question, units were an essential part of the answer. In general there was little premature approximation and only a very few candidates gave their answers to 2 significant figures, thereby losing an accuracy mark. There was no indication of candidates being short of time, and almost everybody attempted all the questions.

## Comments on specific questions

### Question 1

This question was meant to be a straightforward first question but proved to be one of the least well attempted in the whole paper. It was common for a mean of –1.25 to be given as the answer. Candidates clearly had not appreciated that time to do a crossword cannot be negative. Those candidates who expanded the brackets and calculated $\sum t = 405$ and $\sum t^2 = 13732.23$ were usually successful. It was nice to see Var($t$ – 35) = Var($t$) mentioned and more candidates obtained a correct standard deviation then obtained a correct mean.

*Answers*: 33.8 minutes, 2.3 minutes

### Question 2

This was answered well with the majority of candidates gaining full marks. There were a few calculator problems in part **(ii)** with candidates trying to evaluate $\frac{1/2}{4/5}$ as 1 2 4 5 and not using brackets. Some probabilities greater than 1 were seen and using these failed to score the method mark as well as losing the accuracy mark.

*Answers*: **(i)** 0.8; **(ii)** 0.625.

### Question 3

Along with **Question 1** this question was poorly attempted. In part **(i)** the correct z-value was usually seen but many candidates did not appear to use the critical values for the normal distribution, which appear at the foot of the normal distribution tables. Values of z between 1.28 and 1.282 were acceptable, but some candidates used z-values of 1.286, 1.29 etc. A majority of the candidates used +1.282 instead of –1.282. A diagram would have helped. Part **(ii)** was a discriminator question and tested candidates' thinking skills. It was pleasing to find a few candidates who knew that approximately $\frac{2}{3}$ or 67% or 68% of data is within one standard deviation of the mean, and these candidates obtained 2 out of 3.

*Answers*: **(a)** 7.24; **(b)** 546.

**Question 4**

This question undoubtedly was found to be the easiest on the paper. Almost all candidates appeared to know what a stem-and-leaf diagram was, though some did not know what a back-to-back stem-and-leaf was. Since the question did not stipulate that the diagram should be ordered, unordered diagrams gained full marks. There were a number of variations with decimals in the leaves, which were not given marks. Only a very few candidates gave the key both ways and with minutes. A title was also desirable but not seen in many cases.

*Answer*: **(ii)** 15.6 minutes.

**Question 5**

This permutations and combinations question was better attempted overall than in the past. Many candidates obtained full marks for parts **(i)** and **(ii)** and the strong ones attempted part **(iii)** successfully. Again, there were problems with evaluating multiple divisions on the calculator with $\frac{12!}{4!2!}$ being evaluated as 12!    4!    2!. Use of brackets, for example, would have solved this problem.

*Answers*: **(i)(a)** 9 979 200, **(b)** 181 440; **(ii)** 15.

**Question 6**

Many candidates answered this question well. There were still too many candidates who thought 'at least 3' meant exactly 3, or fewer than 3, or more than 3. In part **(ii)** premature approximation of $\frac{1}{7}$ to 2 decimal places, the z-value of –0.1909 to –0.19, and other rounding errors often resulted in the final mark being lost.

*Answers*: **(i)** 0.365; **(ii)** 0.576.

**Question 7**

The quality of answers was variable. There was a large number of candidates who changed to replacement in parts **(ii)** and part **(iii)** and sometimes even started with replacement in part **(i)**. They could not get the required answer given in part **(ii)**, but credit was given for knowing how to evaluate options and fill in the table. A minority of candidates used the permutations and combinations method, the rest used tree diagrams and multiplied probabilities, but many forgot to multiply by the number of options, or only found some of them.

*Answers*: **(i)** $\frac{3}{11}$; **(iii)**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{14}{55}$ | $\frac{28}{55}$ | $\frac{12}{55}$ | $\frac{1}{55}$ |

# MATHEMATICS

**Paper 9709/07**

**Paper 7**

## General comments

Overall, this proved to be an accessible paper. There were no questions that could be identified as being particularly problematic, and candidates were able to demonstrate and apply their knowledge throughout the paper. There was a good spread of marks, with only very few candidates who appeared to be totally unprepared for the examination. There were many good scripts.

**Question 7** was particularly well attempted, apart from the sketch in part **(i)**. **Question 6(i)** was also well attempted, though **Question 6(iii)** proved to be more demanding. **Question 5(i)** was not well attempted, even by more able candidates, with many answers not being given in context. This is a particular problem in this type of question with candidates unable to move from textbook definitions to the question context.

In general, work was well presented with methods and working clearly shown.

It was pleasing to note that, although some marks were lost by candidates due to premature approximation and inability to successfully round answers to three significant figures, this was not as prevalent as in the past. Timing did not appear to be a problem, with most candidates offering solutions to all questions.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, there were also many very good and complete answers.

## Comments on specific questions

### Question 1

Overall this was not a particularly well attempted question. Candidates who found the correct distribution N(1.5, 1.275) were usually able to go on to score well, apart for incorrect attempts at a continuity correction, for instance use of 0.5, 0.05 or –0.05 as the continuity correction. Full marks were available in this question for an answer with the correct continuity correction, but also here for non-inclusion of a continuity correction. Some candidates attempted the question using the total of the 50 observations, and in this case were more likely to apply a correct continuity correction.

*Answer*: 0.713 or 0.714 with continuity correction, 0.734 without continuity correction.

### Question 2

This was quite a well attempted question. It was pleasing to note that relatively few candidates made the usually common error of calculating the variance as $60^2 \times 1.2^2$ rather than $60 \times 1.2^2$. The alternative method of using N(3.2, $\frac{1.2}{\sqrt{60}}$) was seen and credited accordingly.

*Answer*: 0.195 .

**Question 3**

Not all candidates found the setting up of the null and alternative hypotheses straightforward. Errors included using a one-tail test, stating $H_0$ as $\mu$ = 21.7 or even just $H_0$ = 22. When candidates calculated the test statistic further common errors were noted. Most commonly seen were $\frac{22-21.7}{\sqrt{0.19}}$ and $\frac{22-21.7}{\frac{0.19}{\sqrt{8}}}$. A problem noted by Examiners, and commented upon in this report on various occasions in the past, came from candidates' lack of rigour in their comparison of the test statistic and the critical value. Successful candidates either wrote an inequality or clearly showed the values on a diagram in order to draw the correct conclusion.

*Answer*: Not enough evidence to say that the mean has changed.

**Question 4**

This was not, overall, a particularly well attempted question. However, candidates who realised a binomial distribution was required made a better attempt at type I and type II errors than has been the case in the past.. A common error was to identify the wrong probabilities in parts **(i)** and **(iii)** and calculate 1 – P(0,1) in part **(i)** and P(0,1) in part **(iii)**. In part **(iii)** use of 0.2 and 0.8 was occasionally seen instead of 0.09 and 0.91. Many candidates correctly followed through their answer from part **(i)** into part **(ii)**. Use of incorrect distributions (normal and Poisson) were seen, and weaker candidates often made little attempt at the question.

*Answers*: **(i)** 0.0480; **(ii)** 0.0480; **(iii)** 0.601 .

**Question 5**

Few candidates were able to successfully explain why a Poisson distribution may be valid in this particular case. Some candidates were able to quote general conditions for a Poisson distribution, and others made comments relating to time intervals but were unable to clearly express the idea of an average uniform rate for occurrence of the phone calls. Even the most capable of candidates often did not score well on this part of the question. Parts **(ii)** and **(iii)** were, however, better attempted. Most candidates successfully calculated P(8) with $\lambda$ = 10, though the most common error was to use $\lambda$ = 20. Use of the correct N(240, 240) was again often seen in part **(iii)** though errors in finding P($X$ = 250) were common. Most candidates merely standardised with 250 rather than both 249.5 and 250.5.

*Answers*: **(i)** People call randomly, independently, at an average uniform rate; **(ii)** 0.113; **(iii)** 0.0211 .

**Question 6**

Part **(i)** was particularly well answered. The only common errors noted were to calculate the biased estimate for the variance, rather than the unbiased estimate, and the usual error of incorrect substitution into formulae – possibly caused by confusion between different methods of calculation. Calculation of the confidence interval was also well attempted, but part **(iii)** caused problems for some candidates. For those who were able to make a good start to this part, errors included use of an incorrect $z$-value and the appearance of unnecessary factors of two.

*Answers*: **(i)** 1050, 2304; **(ii)** (1030, 1070); **(iii)** 246.

**Question 7**

This question produced good responses, even by weaker candidates. The sketch of the probability distribution was arguably the least well attempted part of the question. Parts **(ii)** and **(iii)** were particularly well attempted by the majority of candidates, though Examiners noted occasions when candidates omitted the essential working required to 'show that'. Part **(iv)** was also quite well attempted, though many attempts at standardisation were seen.

*Answer*: **(iv)** 0.822 .